On facial parity edge colorings

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Facial edge colorings

edge colorings $\rightarrow$ facial edge colorings
(restrictions given by the faces of an embedding to which an edge belongs)

- proper (4 colors),
- non-repetitive (8 colors),
- distance or k-facial $(3k+1?)$.

Motivation

Definition 1 (Bunde et al., 2007)

A *parity edge coloring* of a graph is a coloring of edges, where on every (nontrivial) path at least one color appears odd times.

- *parity chromatic index*, $p(G)$ - the minimum number of colors needed for a parity edge coloring
- proper edge coloring $\Rightarrow \Delta(G) \leq \chi'(G) \leq p(G)$
- $p(G) \geq \lceil \log_2 |V(G)| \rceil$ with equality for paths and even cycles

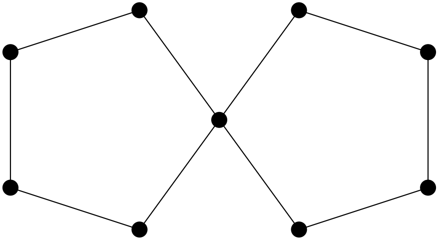
Definition

Definition 2 (Czap et al., 2011)

A *facial parity edge coloring* of a connected bridgeless plane graph is a facially proper edge coloring in which for each face f and each color c, either no edge or an odd number of edges incident to f is coloured with c.

The minimum number of colors needed for a facial parity edge coloring of a graph G is *facial parity chromatic index*, $\chi_{fp}(G)$.

In 2012, Czap presented a graph with $\chi_{fp}(G) = 10$.



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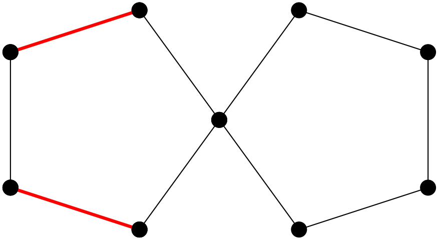

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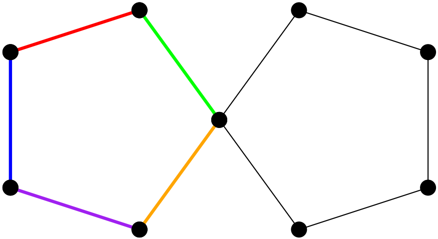

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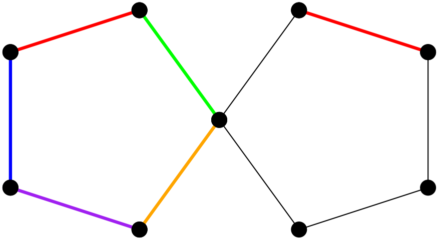

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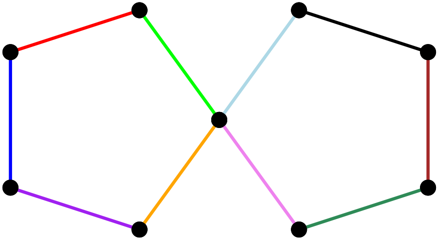

Theorem 3 (Czap et al., 2011)

For every bridgeless plane graph G it holds that

$$\chi_{fp}(G) \leq 92\,.$$

Proved using the discharging method.

Second approach:

- color the edges facially proper;
- divide each color class into subclasses to fulfill the parity condition.

Facially proper edge coloring needs 4 colors (vertex coloring of the medial graph)

Odd subgraphs

Definition 4

An *odd subgraph* of a graph is a subgraph where all the vertices have odd or zero degree.

Theorem 5 (Pyber, 1991)

The edges of every simple graph can be covered by 4 *edge disjoint odd subgraphs.*

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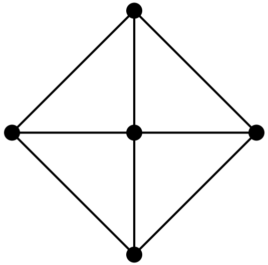

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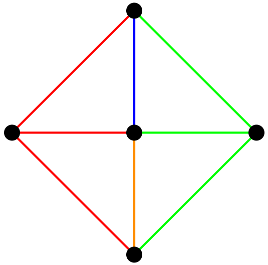

Theorem 6

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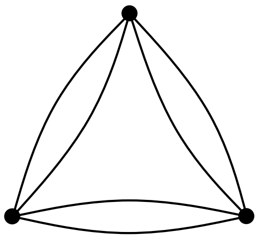
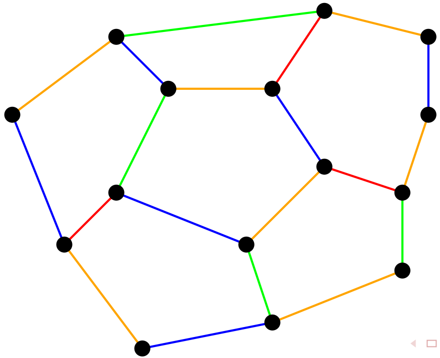
The only graph that needs 6 odd subgraphs:



Second approach:

- color the edges facially proper;
- for each color c create dual D_c restricted to c
- partition D_c into odd subgraphs



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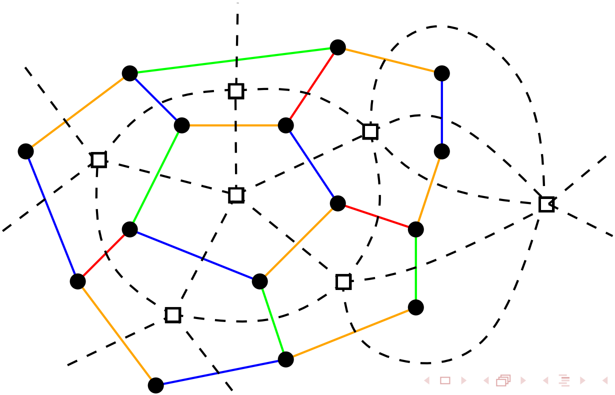

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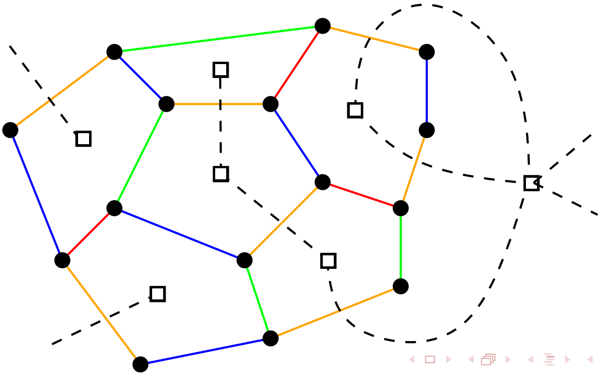

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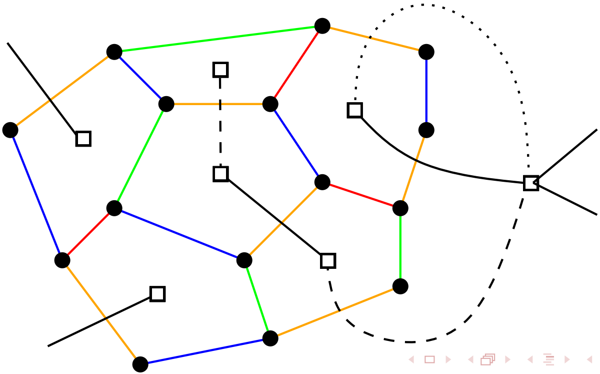

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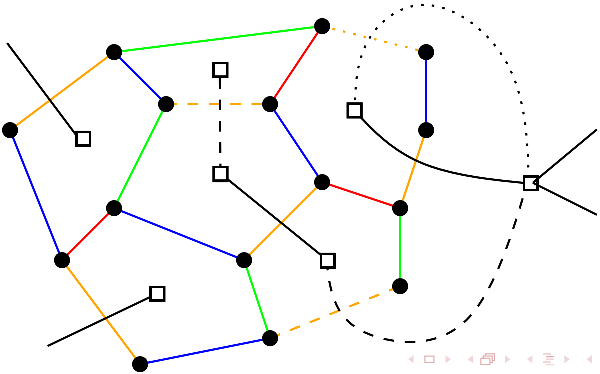

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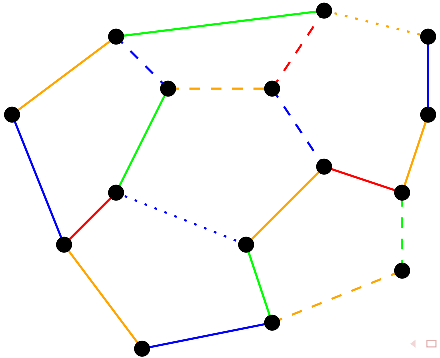

Second approach:

- color the edges facially proper $\rightarrow 4$ **colors**
- for each color c create dual D_c restricted to c
- partition D_c into odd subgraphs $\rightarrow \leq 6$ **colors**

Theorem 7

For every bridgeless plane graph G it holds that

$$\chi_{fp}(G) \leq 24\,.$$

Lemma 8 (Czap et al., 2012)

Every connected plane graph admits a proper facial coloring of edges with at most 5 colors such that for every color the number of common edges between any pair of faces colored by c is odd or 0.

Avoiding multigraphs to use Theorem 5 (since pairs of parallel edges are reducible)

Theorem 9 (Czap et al., 2012)

For every bridgeless plane graph G it holds that

$$\chi_{fp}(G) \leq 20\,.$$

Lemma 10

Every connected plane graph admits a proper facial coloring of edges with at most 4 colors such that for every color the number of common edges between any pair of faces colored by c is odd or 0, unless the common edges of the pair form C_5. In that case, we use precisely one color twice.

Theorem 11

For every bridgeless plane graph G it holds that

$$\chi_{fp}(G) \leq 16\,.$$

Other results

Theorem 12 (Czap, 2012)

For every bridgeless outerplane graph G it holds that

$$\chi_{fp}(G) \leq 15\,.$$

Theorem 13 (Czap, 2012)

For every bridgeless cactus graph G it holds that

$$\chi_{fp}(G) \leq 10\,.$$

Theorem 14 (Czap et al., 2012)

For every k-edge-connected plane graph G it holds that

- $\chi_{fp}(G) \leq 12$, *if* $k = 3$;
- $\chi_{fp}(G) \leq 9$, *if* $k = 4$.

Thank you!